We will correct and save our homework today .....

2.49

constraint:

$g(a,b,c) = abc = V$ ← some constant.

function: $f(a,b,c) = 1.5ab + 2bc + 2ac$

to be minimize

$a,b,c > 0$

Lagrange mult. eqns.

$$\left.\begin{aligned} 1.5b + 2c &= \lambda bc \\ 1.5a + 2c &= \lambda ac \\ 2b + 2a &= \lambda ab \\ abc &= V \end{aligned}\right\} \rightarrow \begin{aligned} b &= a \\ c &= \tfrac{3}{4}a \end{aligned}$$

$$a = \sqrt[3]{\frac{4V}{3}}$$
$b =$ "
$c = \frac{3}{4}$ "

abs. min

Key info:

As $a, b,$ or $c \to 0$

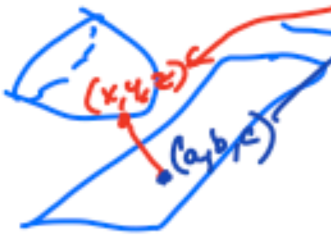
$bc \to \infty$ $ac \to \infty$ $ab \to \infty$

$f(a,b,c) \to \infty$ as (a,b,c) goes to the edges of domain

domain $abc = V,\ a,b,c > 0.$

2.46 Distance between $x = 4y^2 + z^2 + 2$

& $-x + y + 2z = 10$



function to minimize

$$D(x,y,z,a,b,c) = (x-a)^2 + (y-b)^2 + (z-c)^2$$

distance^2 between (x,y,z) & (a,b,c).

Constraints:

$$g_1(x,y,z,a,b,c) = x - 4y^2 - z^2 = 2$$

$$g_2(x,y,z,a,b,c) = -a + b + 2c = 10$$

$$\nabla D = \lambda_1 \nabla g_1 + \lambda_2 \nabla g_2$$

$$\begin{array}{l|l} 2(x-a) = \lambda_1 & +2(x-a) = \lambda_2(+1) \\ 2(y-b) = \lambda_1(-8y) & +2(y-b) = \lambda_2(-1) \\ 2(z-c) = \lambda_1(-2z) & +2(z-c) = \lambda_2(-2) \end{array}$$

$$\lambda_1 = \lambda_2 \Rightarrow y = \frac{1}{8},\ z = 1$$

$$\hookrightarrow x = 2 + 4y^2 + z^2 = \ldots = \frac{49}{16} = 3.0625$$

Plug back into equations

equ with a & b
a & c } plug back into $-a + b + 2c = 10$

$$\ldots \Rightarrow (a,b,c) = \left(\frac{119}{96}, \frac{187}{96}, \frac{223}{48}\right)$$

$\approx (1.2326, 1.9479, 4.6458)$

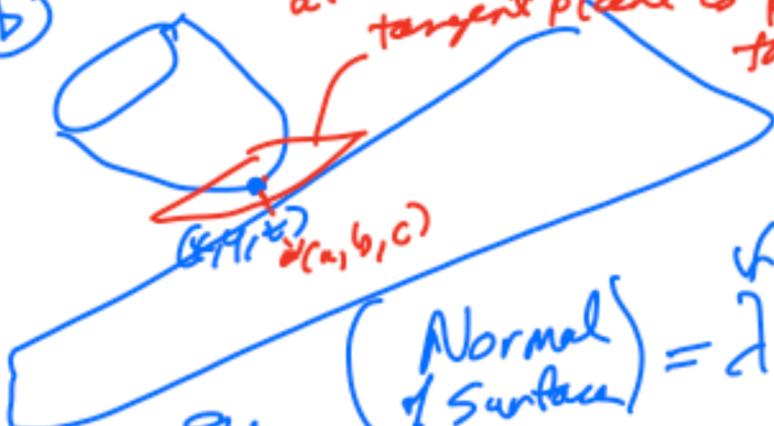
distance $\left\| \left(\frac{49}{16}, \frac{1}{8}, 1\right) - (\quad '' \quad) \right\|$

$\overset{''}{\sqrt{D(\ldots)}} = \frac{175}{96}\sqrt{6} \approx 4.465.$

The closest two points must be a critical pt, so our answer must be the abs. min!

(b)

at closest pt — tangent plane is parallel to the given plane.



constant

$\begin{pmatrix}\text{Normal}\\ \text{of surface}\end{pmatrix} = \lambda \begin{pmatrix}\text{Normal}\\ \text{of plane}\end{pmatrix}$

$\overbrace{x - 4y^2 - z^2}^{g_1} = 2$ surface

$\nabla g_1 = \lambda \nabla g_2$

$\underbrace{-x + y + 2z}_{g_2} = 10$ plane

$(+1, -8y, -2z) = \lambda(-1, 1, 2)$

$\vdots$ $\qquad \lambda = -1$

$$\longrightarrow (x,y,z) = \left(\frac{49}{16}, \frac{1}{8}, 1\right)$$

$$(a,b,c) - (x,y,z) = k(-1,1,2)$$

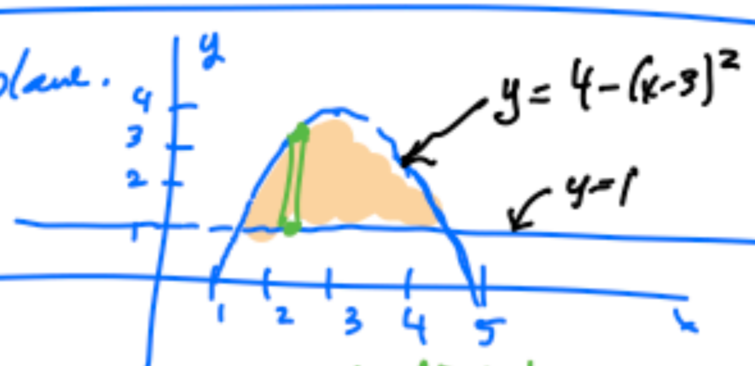
$$-a+b+2c = 10$$

$\Longrightarrow$ $(a,b,c) =$ same as before.

Back to our double integral example:

Example Find the volume under $z = xy + 20$ over the region in the xy plane between $y = 1$ and $y = 4 - (x-3)^2$.

Picture: xy plane.



vertical strip

$$\text{Volume} = \int_{x=}^{x=} \left(\int_{y=1}^{y=4-(x-3)^2} xy + 20 \, dy \right) dx$$

intersection points: $4 - (x-3)^2 = 1$

$$3 = (x-3)^2$$

$$\pm\sqrt{3} = x - 3$$

$$x = 3 \pm \sqrt{3}.$$

$$\text{Volume} = \int_{x=3-\sqrt{3}}^{x=3+\sqrt{3}} \left(\int_{y=1}^{4-(x-3)^2} xy+20 \, dy \right) dx$$

$$= \int_{x=3-\sqrt{3}}^{x=3+\sqrt{3}} \left(\left(\frac{xy^2}{2} + 20y \right) \Bigg|_{1}^{4-(x-3)^2} \right) dx$$

$$= \int_{3-\sqrt{3}}^{3+\sqrt{3}} \left[\frac{x}{2}\left(4-(x-3)^2\right)^2 + 20\left(4-(x-3)^2\right) - \left(\frac{x}{2}+20\right) \right] dx$$

$= \text{sagemath}$

$$= \boxed{\frac{532\sqrt{3}}{5}}$$